Learning yacht hull adjectives and their relationship with hull surface geometry using GMDH-type neural networks for human oriented smart design

Kemal Mert Dogan*, Erkan Gunpinar

Istanbul Technical University, Turkey

ARTICLE INFO

Keywords:
Smart design
Parametric yacht hull design
Adjective based design
Computer aided design
Group Method of Data Handling (GMDH)

ABSTRACT

Several efforts have been made to describe cars by adjectives, however there is no particular work exploring adjectives describing yacht hulls which motivates this study. First, a novel design schema is developed for representing yacht hulls in 3D, which is parametric based and includes several geometric parameters quantifying the hull models. Three surveys are then conducted to learn the relationship between hulls and hull adjectives. In the first survey, yacht models with different geometries are shown to participants to form the hull adjective dictionary. Next, geometric parameters having no impact on the hull adjectives are eliminated via the second survey. Hull adjectives are then matched with yacht hull models via the third survey. The models shown in the third survey are obtained by performing sampling using the Taguchi experimental method. Finally, GMDH-type neural network (GMDH) is applied to the data sets obtained from the third survey to determine the relationships between the hull adjectives and the geometric parameters. GMDH provides mathematical models for each adjective consisting of geometric parameters with coefficients. With the outcomes of this work, we expect that communication between designers and customers can be easier, and adjective based design variations of yacht hulls can be achieved in a shorter amount of time.

1. Introduction

Today's market competition makes it compulsory to develop more appealing designs for customers. The success of a product is not only related to its performance but also to customer perception (Sharma et al., 2016). However, it is nontrivial to quantify human perception due to difficulties of catching human aesthetic shape understandings. With conventional methods, companies need to design diverse products to identify the most appealing designs, which is time-consuming and expensive. These necessities open a new research field about customer oriented designs that aims to capture human feelings about products in order to reduce the number of product trials. Customer oriented design is designated by many other names in literature such as delight design, preference oriented design, smart design and intelligent design (Heinz, 2012; Gangurde and Akarte, 2013; Abelow, 1999). One important study on exploring and adding human factors to the design stages is the work of Chaudhuri et al. (2013), which inspired us to learn adjectives that can be used to express yacht hulls. They suggest assembly based modeling with high-level attributes (or adjectives) that are learned via surveys.

Our study suggests an adjective based design concept for yacht hulls, where hulls are expressed by adjectives. Hull adjectives (such as strong, speedy) can be powerful in terms of representing hull shapes and can serve as a kind of language that can be used jointly by yacht designers and customers. We believe that utilizing such language can facilitate communication between them as well. This concept can also enable designers to make more satisfactory yacht hull designs for customers. In order to explore relationship between hull adjectives and hulls, parametrization of hull shapes is a method that enables us to quantify hulls. By doing this, hull adjectives can be matched with hull shapes. Therefore, we first define a design framework to define hull shapes, where a hull is divided into three sections: Entrance, Middle-body and Run sections. These sections are separately represented with surfaces. Geometric parameters are introduced on these sections and represent important parameters of the hull. Therefore, our objective is narrowed down to determining the relationship between hull adjectives and hull geometric parameters. With the help of three surveys, data is collected, which will be used to determine this relationship. GMDH-type neural network (GMDH) (Kondo, 1998) is utilized to create mathematical models, which
is a function that links geometric parameters with hull adjectives. This paper is organized as follows: Section 2 summarizes the extant literature. Section 3 explains the proposed approach by elaborating on the design framework for yacht hulls, survey methods and GMDH-type neural network. The results and discussion are provided in Section 4. Section 5 concludes this study and provides directions for future research.

2. Related works

Our work relates to several research areas, including yacht hull design, generative design and human oriented design. Related studies will be summarized in this literature review section.

2.1. Yacht hull design

There are many hull design techniques which are recommended in the literature as serving a variety of purposes. Mancuso (2006) suggests an algorithm to generate hulls automatically. This algorithm first defines keel and water lines, canoe body draft, stem angle and some dimensional coefficients with design parameters. Fair B-spline surfaces of a hull are then obtained. Shamsuddin et al. (2006) present a novel non-uniform rational B-splines (NURBS) parametrization method that reduces the number of control points to generate fair and smooth hull surfaces. Perez (2014) proposes a parametric hull design method where hull surfaces are decomposed into three boundary curves: center, chine and sheer lines. These curves are defined with geometric parameters and constraints which are directly related with the geometric hull features. Zhang et al. (2014) suggest a hull design method providing continual functions to define fair ship construction and section boundary lines releasing the errors caused by interpolations and without dealing with complex design parameters. They use the analytic functions to obtain basic structures of section lines by defining logistic and Lame curves. Kim et al. (2016) also cover parametric hull design methods integrated with two type optimization algorithms, which are sequential quadratic programming and particle swarm optimization. In their study, hulls can be modified by the hull modification operators.

2.2. Generative design

Generative design is an important research field, which can turn a computer into a creative design engine. CAD designers can make use of it, and different CAD models in the design space can be automatically generated. Krish (2011) proposes a generative design method for flexible and rapid creative design explorations. Their method is to first build a genotype with some design variables, whose values are changed randomly to generate different designs. The generated designs are then filtered by taking into account some performance criteria (such as manufacturability and cost) to produce feasible designs. Kalogerakis et al. (2012) demonstrate a method that combines design components that form the whole design model to generate creative designs. They benefit from probabilistic relationships between design components to execute the combination process. Shea et al. (2005) define eFFORM, which provides a structural design system to support designers in obtaining feasible designs and a modeling system (Generative Compo- nents) that uses XML models. Kielarova et al. (2015) suggest a computer based generative design method where design evaluation is shaped by user preferences. In their method, creativity and productivity are provided by a design evaluation algorithm and interactive shape grammar, and the variety is provided by genetic operators. Gavacov et al. (2014) use computer aided generative design method for car components to obtain prototypes easily and rapidly. They develop an algorithm that controls processes and includes mathematical shape formulas to obtain bends and curves of car body surfaces; and this algorithm enables to determined required constraints for car design automatically. Cui and Tang (2014) describe a novel representation method that supports generative product design system through flexible shape designs. In their demonstration method, real forming processes and geometric/topological attributes of products are included to simulate these designs dynamically.

2.3. Customer oriented product design

Today’s market conditions make it mandatory to create more attractive product designs that sell easily, which opens new areas for research such as customer oriented product designs. Orsborn et al. (2009) show a way to predict customer preferences as utility functions as well as represent the product designs quantitatively. According to Orsborn et al. (2009), various and innovative product designs can be generated having higher utilities. Kim et al. (2014) suggest a design method including artificial market and a genetic algorithm to a simulate Brand-level diffusion process that examines the impacts of product attributes on market shares. In this way, product designs can be selected among alternatives according to market responses that are predicted by their method. Petitot and Dagher (2011) propose a method to integrate customer preferences to the product design processes illustrating with car’s headlights. A headlight is defined by two Bezier curves and various headlights are generated by manipulating the control points of the curves. Multidimensional scaling (MDS) between design variables are utilized to create design space covering variations of shape features. Finally, customer preferences are related with the design variables defined on the headlight model.

The study of Chaudhuri et al. (2013) is more related to our project’s intent. Using and analyzing a survey, they learn and rank high level adjectives, which are then matched with previously designed assembly parts. A 3D CAD tool is also developed where some components (such as head and body) and adjectives are selected via its interface and new assembly models are generated by connecting the selected components. However, our work is different from that of Chaudhuri et al. (2013). First, parametric representation is used to represent models, whereas mesh models are utilized in their work. The relationships between models and adjectives are found using geometric parameters in our work, while Chaudhuri et al. (2013) compare components one by one to find their adjective degree. Second, Chaudhuri et al. (2013) use existing models in the database, but we perform model sampling using the Taguchi experimental method (Taguchi, 1986) to obtain uniformly sampled yacht hull models in the design space. Third, the proposed technique of Chaudhuri et al. (2013) work with models that are easy to segment. However, a yacht hull is difficult to segment as it consists of smooth surfaces.

3. Proposed approach

The objective of this work is to find suitable adjectives for yacht hulls and learn the relationship between hull adjectives and yacht hulls. To do this, a set of approaches are proposed, which is shown in Fig. 1. First, hull is parametrized by defining its shape using geometric parameters, which will enable us to quantify hull shapes. In the second step, yacht hull adjectives (hull adjectives) are learned via Survey 1. Geometric parameters having no relationship with the learned adjectives are then eliminated in the next step via Survey 2 in order to reduce the number of geometric parameters, which will facilitate the work in Survey 3. In the next step, yacht hulls generated systematically using a parametric approach are inquired of participants via Survey 3 twice and are then matched with hull adjectives. First implementation of Survey 3 has been considered as preliminary study and its output is used to determine further arrangement for adjectives and final interface of Survey 3. Data collected via second implementation of Survey 3 is utilized to learn relationships between geometric parameters and hull adjectives.

3.1. Yacht hull parameterisation

Learning the relationship between yacht hulls and hull adjectives is difficult without quantifying the hull. For this reason, hulls are
parametrized by defining their shape using geometric parameters, which are defined on the 3D hull model. To represent this 3D model, a good representation schema should be utilized, which will be introduced in the next subsection.

3.1.1. Design framework

In this step, design framework for 3D representation of yacht hull models will be detailed. The following two issues are considered while designing the design framework:

- **Generation of different hull shapes**: Yacht hulls with variable geometries should be produced as much as possible. This enables us to learn hull adjectives in the hull design space effectively. With less variety of hull shapes, it may not be possible to learn these adjectives. The proposed design schema can produce displacement hulls such as arched and rounded, semi-displacement hull, planing hulls such as double chine Vee shape and flat bottom.

- **Detection easiness of hull adjectives**: The parametric design method is a good way to quantize the yacht hulls. Geometric parameters on the 3D yacht hull model can be defined. By utilizing these parameters, learning the relationship between hull adjectives and yacht hull shapes can be easier.

To develop a new design framework, yacht hulls with different geometries are first collected. Important hull features (such as hull length) in these collected hull models contributing hull's geometric appearance are then found. Design framework is developed while considering these hull features in the collected models as much as possible. Next, the yacht hull is divided into three sections Entrance, Middle-body and Run (see Fig. 2 (a)) in order to easily learn relationship between hull adjectives and hull models. For example, if the speedy adjective is more related to the length of the Entrance section, it can be hard to understand this relationship without hull separation in the design framework. Furthermore, this separation allows designers quick sectional modifications.

Fig. 2 (a) shows the surface model of a yacht hull with its divided sections. Surfaces of these three sections are generated first and then merged in order to obtain the whole surface model. G0 and G1 geometric continuities exist between surfaces of neighboring sections. The surface of each section is obtained by performing loft operation using characteristic curves, which are shown in Fig. 2 (b). Generation of surface models for each section is explained in the below:

- **Entrance Section**: As shown in the left image of Fig. 2 (b), Entrance section is composed of five characteristic curves. These are top guide, bottom guide, forward profile, first station profile and feature guide. Loft operation is conducted between forward profile and station profile. Top, bottom and feature guide curves serve as guides for the loft operation.

- **Middle-body Section**: Characteristic curves of this section are top guide, bottom guide, first station profile, second station profile and feature guide. Loft operation is performed between the first and second stations. Top, bottom and feature guide curves are used as guides for the loft operation.

- **Run Section**: The characteristic curves of the Run section are top guide, bottom guide, second station profile, third station profile and feature guide. The surface is obtained between the second and third station profiles by setting top, bottom and feature guide curves as guides.

Note that only half of the hull is obtained after the loft operation, and mirroring operation can transpose the half body into a complete...
hull body.

All characteristic curves are represented using cubic Bezier curves except forward profile and some portion of station profile. A station profile consists of curves and lines. Fig. 2 (c) shows a station profile, which has upper and lower station curves represented using Bezier curves, thickness line for the feature guide and width line for the bottom panel represented using straight lines. The number of chines in a hull can be determined by arranging the \( a_2 \) parameter. And hull with double chines, hull with single chine and hull without chines can be generated. Double chine hull is generated by setting the \( a_2 \) parameter to non-zero values. If the \( a_2 \) parameter takes zero value, hull with single chine or without chine can be generated. If G1 continuity between upper and lower station curves exists, hull does not have a chine. Otherwise, it is a single chine hull.

3.1.2. Geometric parameters

In this section, geometric parameters will be introduced that are defined for characteristic curves. Thirty-eight geometric parameters are introduced in total, which seems important to us and can comprehensively represent the yacht hulls altogether. These parameters will be utilized to find the relationship between yacht hulls and adjectives. The number of these parameters can be increased if essential. Also we keep values of the \( a_1 \) and \( a_2 \) parameters constant along the hull to simplify our study. According our observations, they are mostly taken as constant in hull designs. Fig. 4 shows a tree consisting of characteristic curves of each section with related geometric parameters. Note that the station profile \( SP^b \) belongs to both Entrance and Middle-body sections as it used as a profile for the loft operation in the Entrance and Middle-body sections. Similarly, \( SP^m \) belongs to the Middle-body and Run sections.

Fig. 5 shows geometric parameters and their definitions. Parameters on Entrance section, Middle-body section, Run section and the station profile are indicated separately. There are three types of geometric parameters: Linear, angular and unitless parameters. \( L^1, B^0, W^4, D^4, H, a_1, a_2 \) are linear in meters. \( \alpha, \beta, \theta, \alpha_2 \) are angular in degree. \( R_x^0, R_x^1, R_x^2, R_y^0 \) are unitless parameters. Some of the parameters have \( x \) over their symbols, where \( x \) can be \( e, m \) and \( r \) standing for Entrance, Middle-body and Run sections respectively. For example, \( L^e \) refers to the length of Entrance section, \( L^m \) is length of Middle-body section and \( L^r \) is used for the length of Run section. The minimum radius of the curvature parameter is defined for characteristic curves. Instead of using a dimension unit for this parameter, three levels are introduced to define curve shapes (see Fig. 3). The level 1 represents straight shapes whose minimum radius of curvature values are higher than 200 m. The level 2 is for rounded shapes whose minimum radius of curvature values range between 90 m and 200 m. The level 3 represents a less curved shapes with minimum radius of curvature values less than 90 m. We restrict minimum radius of curvature to get discrete values as small changes in this parameter is hard to be perceived by users.

3.2. Learning relationships between yacht hulls and hull adjectives

This section provides a set of approaches to learn relationships between yacht hulls and hull adjectives. Hull adjectives are first learned via Survey 1. Some geometric parameters are then eliminated via Survey 2. Via Survey 3, data is collected to identify relationships between geometric parameters and hull adjectives. Next, sub sections will elaborate all these approaches.

3.2.1. Learning hull adjectives via survey 1

In this step, hull adjectives expressing yacht hull shapes are explored. Three main criteria are considered to determine the adjectives:

- The adjectives should be frequently used by people.
- They should be less relativistic and less related to the predefined geometric parameters.
- The adjectives all must have different meaning from each other to prevent confusion for subjects.

There are several ways to learn hull adjectives such as survey and social listening. By means of Survey 1, a variety of adjectives were collected from people. Fig. 6 (a) shows a question from Survey 1. Survey participants are asked to express 3D yacht hull models having a variety of geometries that are collected from the net, by an adjective. This survey was applied to 97 participants with ten different hull models. Most of the adjectives were learned via Survey 1, some of which were eliminated later. We selected some of the adjectives to use in the next surveys based on the second above-mentioned criterion. For example, the adjectives bad, ugly, good are too much relativistic and the adjectives wide, long, thin are directly related to the geometric parameters. Additionally, some adjectives were grouped due to their close meanings. For instance, the adjectives strong, enduring, durable and solid have close meaning, and therefore they were grouped under the strong adjective. Fig. 6 (b) shows adjectives with the usage frequency, which have been determined after the filtration process. 15 most preferred adjectives were used in a preliminary study of Survey 3. The results of this survey showed us that some adjectives were not selected for any model. Furthermore, we used retrospective think aloud (RTA) method (Hyrsykari et al., 2008) where participants were asked to tell reasons for their choices, difficulties that they were faced in the survey. According to participants’ feedbacks, some of the 15 adjectives had still close meanings and therefore, additional grouping was applied for these 15 adjectives. Social listening method was used as well to find adjectives to represent former adjectives with clear ones. Finally, ten hull adjectives were selected that were investigated in the further steps, which are listed as follows: strong, speedy, comfortable, aesthetic, usual (common), aggressive, compact, modern, charismatic, cute. Furthermore, some people had difficulty to make adjective selections for the yacht hulls or they interpreted adjectives completely different from their meaning. To standardize adjective meanings for all participants, the below adjective definitions, which were learned from dictionaries (Cambridge dictionary, 2016; Dictionary, 2016; Oxford dictionary, 2016), were explained them before the second implementation of Survey 3.

- **Strong** \( (A_1) \): Difficult to break, destroy, or able to support a heavy weight or force. Looks powerful and heavy.
- **Speedy** \( (A_2) \): This design type allows the air to move over and around the yacht hull in a smooth manner. Also light weight-looking.
- **Comfortable** \( (A_3) \): Comfortable hull design provides a pleasant feeling and do not give you any physical problems. Relaxed and free from pain.
Exploration process of eliminated and not be used in the further steps to reduce the number of impact on the predefined parameters (see Fig. 7(a)), which is shown in below:

We divided the parameters into two groups as Basic parameters and Sub parameters.

1. **Aesthetic** \( (A_4) \): An aesthetic hull is one that shows great beauty. Relating to the enjoyment or study of beauty. Looks more like curved shape.

2. **Usual (Common)** \( (A_5) \): Normal or used most often. The same in a lot of places or for a lot of people. Typical design.

3. **Aggressive** \( (A_5) \): Angry-looking design. This design makes the yacht looking boldly assertive and forward. Determined to win or succeed and using forceful action.

4. **Compact** \( (A_5) \): Consisting of parts that are positioned together closely or in a tidy way, using very little space.

5. **Modern** \( (A_5) \): Designed using the most recent ideas. Characteristic of present and recent time; contemporary; not antiquated or obsolete.

6. **Charismatic** \( (A_6) \): Exercising a compelling charm which inspires devotion in others. Having, or characteristic of charisma¹.

7. **Spiritual power or personal quality that gives an influence or authority over large numbers of people.**

8. **Cute** \( (A_{10}) \): Attractive in a pretty or endearing way.

### 3.2.2. Elimination of some geometric parameters via survey 2

Some of the geometric parameters which do not have any impact on predetermined 15 hull adjectives are detected and eliminated in this step. We divided the parameters into two groups as Basic parameters and Sub parameters (see Fig. 7(a)), which is shown in below:

- **General form of the hull** is constructed using Basic parameters, which are \( L, L_m, L_r, B, B_m, B_r, D_f, D_m, D_r, D_1, D_2, D_3, a_1, H, \theta, \alpha, \alpha_2 \) and \( \beta \).

- **The detail appearance of the hull** is obtained using Sub parameters, which are \( R_p, R_p^0, R_p^0, R_q, R_q^0, R_q^0, R_r, R_r^0, R_r^0, R_s, R_s^0, R_s^0, W^e, W^m, W^r \) and \( a_2 \).

It is believed that some of the Sub parameters may not have any impact on the predefined hull adjectives, and therefore they should be eliminated and not be used in the further steps to reduce the number of geometric parameters. Such parameter elimination can facilitate the exploration process of finding the relationship between hull adjectives and geometric parameters.

First, a cubical design space (see Fig. 7(b)) is generated whose parametric dimensions are selected as length \( (L) \), beam \( (B) \) and depth \( (D) \) of the hull, since they are the most important Basic parameters. Three parametric levels are obtained for each parametric dimension, which stand for small, medium and large values for \( L, B \) and \( D \) parametric dimensions. These values are assigned using the yacht catalogs of Ege Yat (Ege Yat catalogs, 2016), which is a yacht designing and building company in Turkey. As a result, 27 \( (3^3) \) yacht hull models are sampled in the cubical design space. Only three of these models are utilized, as it is impractical to use all these 27 models. These models reside on one diagonal of the cubical design space, which can be regarded as small, medium and large size of hull models. We call these models Base models (see Fig. 7(b) models enclosed by blue rectangle). Parametric values of 16 Sub parameters are changed one by one on the Base models. Three parametric levels are taken for each Sub parameter where the parameter values are small, medium and large. And therefore, several yacht hull models are obtained, which will be used in the survey. For each Sub parameter, a small size Base model is first shown to survey participants, who are then asked to choose an appropriate hull adjective for this parameter. These adjectives are the 15 hull adjectives determined before the preliminary study in the previous step. Next, new yacht models having the other two parametric levels of the Sub parameter are shown to participants, who are asked whether the selected adjective choice changes or not (see Fig. 8(a)). The same process is performed for the other two Base models. We collect data from 75 participants in two sets with one-week interval in order to make participants focus well on the surveys. Survey data is analyzed for each Base model separately as seen in Fig. 8(b) and percentages for the Changed, Unchanged and Neutral selections are calculated. If percentages higher than 80% is obtained for the Unchanged option for all three Base models, the Sub parameter is eliminated. Otherwise, it is not eliminated. Fig. 8(b) illustrates data distributions for the \( R_p^0 \) and \( R_s^0 \) Sub parameters. \( R_p^0 \) is eliminated as it satisfies the threshold criterion (80% criterion). However, \( R_s^0 \) is not eliminated, because the criterion is not satisfied for all three Base models. As a result of Survey 2, the following nine geometric parameters are eliminated: \( R_p^0, R_q^0, R_q^0, R_r, W^e, W^m, W^r, a_2 \).

### Fig. 4. Geometric parameter tree.
3.2.3. Collecting data for learning relations between geometric parameters and hull adjectives via survey 3

In this part of the study, a data set is prepared to learn the relationship between yacht hulls and geometric parameters. The Taguchi experimental method (Taguchi, 1986; Öztekin et al., 2013; Chang, 2008) is used to do uniform sampling based on geometric parameters for the generation of 3D yacht hull models that will be used in Survey 3. Participants are asked to evaluate these generated hull models according to their level of relation with hull adjectives. Finally, the survey results are arranged and utilized to explore the relationship between yacht hulls and geometric parameters.

3.2.3.1. Sampling by Taguchi experimental method. Twenty-five geometric parameters left after eliminating some of the geometric parameters. When a naive approach with three parametric levels for each geometric parameter is used, 847,288,609,443 (3^{25}) possible yacht hull design variations can be obtained, which is impractical to be used in Survey 3 for participant scoring. Therefore, the Taguchi experimental method is utilized to obtain sampled yacht hull models in the design space. The Taguchi experimental method suggests using orthogonal arrays (Bush, 1952) which can provide uniform sampling from the design space by taking hull samples having more geometric variations as much as possible with a less number of hulls. L54 orthogonal array is chosen for 25 geometric parameters with three parametric levels (1, 2, 3) for each parameter (only two levels for one of the parameters due to restriction of L54 array). 54 yacht hull models are generated, which will be used in Survey 3. Fig. 10(c) shows L54 orthogonal array, in which each column belongs to one geometric parameter and each row represents parametric levels of a sampled hull model. When these parametric levels (see Fig. 9 for the levels L_1, L_2, L_3) are replaced with the corresponding parameter values, geometric parameter values of the sampled hull models are obtained as displayed in Fig. 10(a). For example, \( B^e \) takes the parametric level 1 in the first row as shown in Fig. 10(c), which refers to the parameter value 1.2 m.

Although reducing the number of hull models is quite advantageous, there are several drawbacks of using orthogonal arrays. Naturally, the whole design space is not covered fully by the sampled models so that learning the relationship between some of the hull adjectives and hulls...
can be hard to achieve. The second drawback is that the orthogonal array can give infeasible hull designs (see Fig. 11(a)) as relationship between geometric parameters (i.e., geometric constraints) is not considered by the Taguchi method. Note that feasibility of a yacht hull model is judged with the help of a designer in a yacht industry just by checking geometric appearance.

In this study, infeasible yacht hull models are manually modified by changing parametric levels of geometric parameters one by one until feasible hull models are obtained (see Fig. 11(b)). Fig. 11(c) shows an illustration of a design space with feasible and infeasible regions, where feasible regions (gray zone) involve feasible hull models (yellow points) and infeasible region (blue zone) involve infeasible hull models (depicted by black points) exist. To obtain a feasible model from an infeasible model, the model should move to the feasible region (as illustrated with arrows in the figure) by modifying some of the geometric parameters. During this modification, the Euclidean distance between the original and modified hull models in the hull design space should be as short as possible. To do this, modifications are controlled by similarity matrices, which are calculated by the Euclidean distance formula (Eq. (2)) (Gigour, 2006). Since the geometric parameters have ranged in different scales, parameter values are standardized using z-score method (Eq. (1)) to refrain from predomination problem before the distance calculation.

\[
d(p, q) = \sqrt{\left(p_1 - q_1\right)^2 + \left(p_2 - q_2\right)^2 + \cdots + \left(p_n - q_n\right)^2}
\]

In Eq. (2), \(d(p, q)\) is a function that computes distance between two hull models in the \(n\) parametric dimensions. \(p_i\) and \(q_i\) represent standardized parametric values of the \(i^{th}\) geometric parameter for the two hull models \(p\) and \(q\), where \(i\) can vary from 1 to 25 (number of geometric parameters). The Taguchi experimental method produces 3 feasible and 51 infeasible hull models. After modifying an infeasible hull model, the parametric distance is checked. Fig. 11(d) displays similarity values for hull models and their corrected (feasible) ones which are computed using Eq. (3). \(d\) denotes the distance between the modified and the initial hull model. \(d_{\text{ave}}\) is the distance between two hull models (generated by the Taguchi experimental method) having maximum distance. Higher similarity value stands for closer hulls in the design space. This value is between 100% and 80% for 23 hull models, 79% and 70% for 27 hull models and 69% and 65% for 4 hull models. Fig. 11(d) shows also a horizontal line with red color indicating the average of similarity values.
Fig. 8. (a) User interface of Survey 2 (b) Survey data distributions for the $R_{5}$ (eliminated) and $R_{6}$ (not eliminated) Sub parameters.

Fig. 9. Initial corresponding parameter values for three levels. Calculated means and standard deviations after modification of the infeasible models.

Fig. 10. (a) Replacing parametric levels with parameter values (b) Standardized parameter values (c) L54 orthogonal array for geometric parameters of yacht hulls.
for model pairs, which is 79.68%. Therefore, it can be claimed that overall modifications are made in an acceptable range to keep uniformity of samples in the design space. As a future work, we plan to study the extended version of the Taguchi experimental method to produce much better uniform sampling for yacht hulls.

Note that standardization for distance calculation was executed with the initial and modified models together, while the data set input for the GMDH algorithm is obtained by standardizing geometric parameters of the 54 feasible models. Fig. 9 depicts the mean $\mu$ and the standard deviations $\sigma$ for each geometric parameter, which are obtained using 54 feasible models.

$$%s = 100 \times \frac{d_{\text{max}} - d}{d_{\text{max}}} \quad (3)$$

3.2.3.2. First implementation of survey 3. Survey 3’s final interface is created according to the first implementation of Survey 3’s results and feedbacks from the participants. In the first implementation, 54 hull models with five viewpoints (front, side, back, top and isometric view) are shown to the 75 participants which are asked to score hull models based on the hull adjectives. The survey consisted of three sets each of which was implemented with one-week interval. Furthermore, all 15 adjectives are asked at a time to give a score between 0 and 10 to the hull models for each adjective. The scores are divided into three levels: High (between 8-10 points), medium (between 5-7 points) and low (between 0-4 points). According to outcome of this survey implementation, some of the adjectives were eliminated and some of them were grouped into a single adjective. Finally, ten adjectives (mentioned in section 3.2.1) have been determined for the second (final) implementation of Survey 3. The number of viewpoints is also reduced as participants got confused when many viewpoints exist on survey interface. Additionally, we observed that participants have difficulties to vote for many adjectives at a time within the scale ranged in 0–10. Therefore, adjectives are divided in two sets and Likert-type scale is instead of 0–10 scale used (see Fig. 12).

3.2.3.3. Second implementation of survey 3. In this section, 54 yacht hull models generated by the Taguchi experimental method will be matched with ten hull adjectives. All yacht hull models shown to survey participants have same colors, same cabin geometry and dimensions. Cabins are always located in the Middle-body section. Such normalization is done in order to allow participants to have a similar degree of focus on the yacht models. Furthermore, yacht hull models are shown at the same viewpoints of front, side and top views. Note that side views are placed first after scaling according to the hull overall lengths. Top and front views are then inserted using orthographic projection rules. By doing this, participants can easily recognize and compare changes in parameter values for different hull models. 3D models of the yacht hulls are also included as a link in Survey 3’s user interface for clear understanding of the models.

As illustrated in Fig. 12, two sets were prepared for Survey 3 where only five out of ten adjectives are asked to each participant. Likert-type scale is used, in which negative (Very poorly and Poorly) and positive (Well, Very Well) scales are placed symmetrically around a midpoint (neither). This scale makes participants easy to score and provides easy interpretation for the survey analysis. Three out of 54 models are selected as control models that are well distributed in the orthogonal array (9th, 27th, 45th models). These models are asked twice through the survey process. To check reliabilities of participant scores, a reliability metric is computed for each participant using Eq. (4.1). $r_{\text{check}}$ represents the reliability metric, $m_1$ is the first score given for the control model, $m_2$ is the second score for the model and c is a variable that takes one of the control model numbers (9, 27, 45). To use this equation, scores are first converted into integer values as following: Very poorly (1), Poorly (2), Neither (3), Well (4) and Very Well (5). Differences of relevant scores are then divided by 5 to normalize the metric. Reliability of a participant is denoted by $r_{\text{check}}$ in Eq. (4.2) and is computed by taking the average of the $r_{\text{check}}$ Values. As a consequence, the data of a participant whose reliability metric is less than %80 is eliminated.

$$r_{\text{check}} = \frac{|m_1 - m_2|}{5} \quad (4.1)$$

$$r_{\text{check}} = 100 - 100 \frac{\sum r_{\text{check}}}{3} \quad (4.2)$$
Each set of Survey 3 was applied to averagely 72 undergraduate students (3rd/4th year students) with one week interval. Before starting the surveys, a brief presentation is made to express purpose of research, introduce the geometric parameters to make them aware about geometric details of the models and describe definitions of the hull adjectives.

3.2.3.4. Arranging data of survey 3. After collecting data from Survey 3, it has to be arranged for the next step, where the relationship between hull adjectives and geometric parameters will be learned. Since the adjectives are independent from each other, data sets are prepared for each adjective separately. Survey results are analyzed using Box and Whisker Plot (box plot in brief) (Frigge et al., 1989), which visually represents

![User interface of Survey 3 of a) Set 1 b) Set 2.](image)

Fig. 12. User interface of Survey 3 of a) Set 1 b) Set 2.

![Box plots for Model 2](image)

(a) Box plots for Model 2 (b) Box plot elements.

Fig. 13. (a) Box plots for Model 2 (b) Box plot elements.
the model is assigned to neither the class "adjective so as to be assigned to the class "class whisker) and belongs to the score "speedy. Therefore, Model 2 is discarded from the data sets of these adjectives. For given for the adjective, the balance point which means that Model 2 is in the class "Neither" -- "poorly" and "very poorly". This model is poorly described by this adjective. By doing these for all hull models (equations). GMDH (Group Method of Data Handling) type neural network does not need much preliminary information to create structure of network such as number of layers, neurons in hidden layers, or thresholds, to pass to the next layer. Instead, it automatically organizes the neural network architecture (Qiu-Min et al., 2013; Ivakhnenko et al., 1994).

\[
Y^*(x_1, \ldots, x_n) = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} a_{ij} x_i x_j + \ldots
\]

Fig. 14 shows the data set for the aesthetic adjective, where the first columns shows the hull number, the next 25 columns represent the standardized values of geometric parameters and the final column shows the assigned class for the hulls. "1" stands for the hull that has the adjective characteristics ("well" and "very well") and "0" is for the hull without the adjective property ("poorly" and "very poorly"). Likert-type scale can be thought as a pair of scales, which has a balance point (Neither (3) and two sides (poorly (1-2) and well (4-5)). The side, having more weight wins. In other words, if the box is placed above the score "3", it represents %75 of the data (sum of the box and the upper whisker) and belongs to the score "well". Therefore, it is assigned to the class "1". If it is below the score "3", this model is poorly described by this adjective so as to be assigned to the class "0". Additionally, if the box shows a distribution equally spreading along both side of the score "3", the model is assigned to neither the class "0" nor the class "1". And therefore, the model is not considered in the data set for the relevant adjective. Fig. 13 depicts the box plots of Model 2 for the strong, speedy, aesthetic, usual and compact adjectives. According to the plots, the scores given for the strong and compact adjectives spread equally, which means that the hull model does not involve in any of two classes (0 and 1). Therefore, Model 2 is discarded from the data sets of these adjectives. For the speedy and aesthetic adjectives, the boxes and overall means are below the balance point which means that Model 2 is in the class "0". Finally, the box and mean is above the balance for the usual (common) adjective, so the model is in class "1" for this adjective. By doing these for all hull models, adjective classes for each yacht hull model are learned. At the end, one data set is prepared for each hull adjective, so 10 data sets in total.

3.3 Learning relationships between geometric parameters and hull adjectives via GMDH-type neural network

In this section, relationships between geometric parameters and hull adjectives will be identified and will be expressed by mathematical models (equations). GMDH (Group Method of Data Handling) type spread of data distribution. A box plot basically consists of quartiles, lower and upper whisker, median, greatest and least values (see Fig. 13). The box portion covers interquartile range (IQR = Q3 – Q1), which means spread of %50 of data is represented by the box, while upper and lower whiskers cover higher or lower %25 of data than the midpoint "3". Median, which indicates the middle data after the data is sorted from smallest to largest, is central point of IQR (see Fig. 13 (b)). Furthermore, this plot shows mean of the data depicted by cross sign (x). Outlier is represented with a dot and the greatest and least scores given excluding outliers are represented with perpendicular lines to the whiskers.

Fig. 14 shows the data set for the aesthetic adjective. The adjective index \( Y \) (in short) denotes adjective index for the \( z^{th} \) hull adjective \( A_z \) where \( z \) is integer varying between 1 and 10. A greater value of the adjective index \( Y^*(x_1, \ldots, x_{25}) \) for a given hull with \( x_1, \ldots, x_{25} \) geometric parameters than a cut off value (threshold) represents a hull that can be expressed by the \( z^{th} \) adjective.

The adjective index \( Y^* \) will be learned for each adjective separately via GMDH. We use k-fold cross-validation for verifying accuracy performance of the obtained mathematical equations. The data set is partitioned into \( k \) sub-groups and cross-validation is executed between these groups. One of the groups are held out as test data and the mathematical models are obtained with \( k - 1 \) sub-groups. This cross-validation process is then repeated \( k \)-times by taking the other sub-groups as test data. During iterations, a criterion value is calculated that refers to error between actual and predicted output value. Mathematical models are generated and updated iteratively until prediction results cannot be improved and the criterion value takes a smaller value. The value for \( k \) is varied between 3 and 5 providing best accuracy and lower criterion value for the mathematical models obtained. GMDH Shell Forecasting Software (GMDH Shell, 2016) is used to learn the mathematical models for the hull adjectives.

4. Results and discussion

In this section, the mathematical models computed in the previous step for the hull adjectives will be outlined. Performance measures will then be used to check reliability of the mathematical models. Finally,
possible improvements of the current work will be discussed.

The mathematical models were obtained for each hull adjective, which can be seen in the Supplementary Material. As we use binary classification, adjective index values greater than 0.5 means greater probability for a model that belongs to the class 1. Otherwise, the model belongs to the class 0, which means that the hull cannot be expressed by that adjective. Following conditions are used to assign hull models to the one of adjective classes (0 and 1):

\[
\text{Class } = \begin{cases} 
1 & Y^2 > 0.5 \\
0 & \text{otherwise}
\end{cases}
\]

For example; the geometric parameters \( L^m, L^n, L^r, L^p, D^m_1, D^m_2, D^p_1, D^p_2, R^m_1, R^m_2, R^p_1, R^p_2 \), \( a, \alpha, \theta \) are the important parameters for the adjective strong (A1) as they exist in its equation. Value of \( Y^2 \) is computed by putting values for these parameters into the obtained mathematical model. Its adjective class (strong or not) can then be determined. The relevant geometric parameters that are involved in the mathematical models of the adjectives are written in below:

- \( A_{22}: L^m, L^n, L^r, B^m, B^r, D^m_1, D^m_2, R^m_1, R^m_2, a, \alpha, \theta \)
- \( A_{23}: L^m, L^n, L^r, B^m, B^r, D^m_1, D^m_2, \beta, \alpha, \theta \)
- \( A_{44}: L^m, L^n, L^r, B^m, B^r, D^m_1, D^m_2, R^m_1, R^m_2, R^p_1, R^p_2, R^n, \alpha, \beta \)
- \( A_{25}: L^m, L^n, L^r, B^m, B^r, D^m_2, \alpha \)
- \( A_{26}: L^m, L^n, L^r, B^m, B^r, D^m_1, R^m_1, R^m_2, a, \alpha, \theta \)
- \( A_{17}: L^m, L^n, L^r, B^m, B^r, D^m_1, D^m_2, D^p_2, R^m_1, R^m_2, R^n, H, a_1, \alpha_2 \)
- \( A_{10}: B^m, D^m_2, R^m_1, R^n, a_1 \)
- \( A_{21}: L^m, L^n, L^r, B^m, B^r, D^m_1, D^m_2, D^p_2, R^m_1, R^m_2, R^n, a, \alpha \)
- \( A_{10}: L^m, L^n, L^r, B^m, B^r, D^m_1, D^m_2, D^p_2, R^m_1, R^m_2, R^n, \theta \)

4.1. Verification of the obtained mathematical models

Four performance measures are utilized in this work: accuracy, \( f \)-measure, criterion value and RMSE. Success of machine learning algorithms is mostly measured with the prediction accuracy. Therefore, successful mathematical models that are created by the GMDH-type neural network should be able to predict a hull adjective for a given hull in high accuracy. The prediction results of the mathematical models for each adjective are shown in Table 1. Accuracy is calculated as the percentage of number of correct predictions among all predictions. Accuracy for the adjectives A1, A2, A4, A5, A6, A7, A8, A9 is %100 as all instances were predicted correctly. One out of 52 was incorrectly classified by the mathematical model for (A3). Therefore, accuracy is %98.1 for the comfortable adjective. Finally, two out of 49 hull instances were incorrectly classified for A10, so the accuracy percentage is %95.9. Root Mean Square Error (RMSE) is also another way to measure accuracy which calculates root of the squared sum of all differences between the predicted and actual classes. RMSE was calculated as 0.137 and 0.202 for A3 and A10, which are acceptable. It is 0 for the remaining mathematical models. On the other hand, criterion value indicates prediction errors calculated during k-fold cross validation. Small criterion value refers well-fitted mathematical model. Criterion value for A1 is 0.19901, A5 is 0.1071, A8 is 0.13982, A3 is 1.41 \times 10^{-6} and 0 for the other mathematical models.

We also check \( f \)-measure, which is another performance measure and harmonic mean of \( precision \) and \( recall \) (Eq. (6)), \( f \)-measure can be thought as a stability of mathematical models that shows accuracies obtained by force or not. Precision shows the rate of correct predictions of hull models among all retrieved models for relevant adjectives, while recall displays a rate of the retrieved correct samples to the total samples for relevant adjective. Since both of them are important measures and \( f \)-measure includes them equally, we analyzed only \( f \)-measure results that should be as possible as close to “1” for mathematical models of high quality. The created mathematical models have respectively low \( f \)-measure values for the A3 and A10 adjectives which are 0.981 and 0.959, however quite sufficient to be accepted.

\[
f_{\text{measure}} = \frac{2 \times \text{precision} \times \text{recall}}{\text{precision} + \text{recall}} \tag{6}
\]

4.2. Adjective based hulls

In this study, parameter variations were obtained by the Taguchi orthogonal array without considering any fairness criterion between hull surfaces, hydrostatic performances or any other criteria needed for the marina structures. An infinite number of hull models satisfying the mathematical form of the desired adjective can be generated. Note that it is crucial to generate hull models both satisfying the mathematical form(s) of the desired adjective(s) and with having good hydrodynamic performance. However, the main objective of this study is to learn relationships between hull adjectives and hull geometric parameters. Adjective-based hull models are given in Fig. 15 with their adjective index values, which were generated using the technique of Khan et al. (2017). Geometric parameter values for these hulls can be seen in the Supplementary Material. Appendix 1 displays the hydrostatic results for the hulls, which were computed using Caeses Software (Caeses, 2017).

4.3. Possible improvements

This research is challenging for several reasons. First of all, adjective understanding for yacht hulls may differ from person to person, which affects the prediction results of this study. The computed mathematical models can even change from one region to another. To define hull adjectives accepted by many yacht designers and customers, it is recommended that the number of survey participants should be increased. By doing this, more generalized results for the mathematical models can be obtained. In this work, 15 hull adjectives are defined at the beginning, but after preliminary study this number was reduced to 10. The mathematical models are learned only for these adjectives. The Taguchi experimental method (Oztekin et al., 2013) is utilized to sample yacht hull models in the yacht hull design space. This is done because using a large number of hull models in surveys is impractical. On the other hand, sampling can make finding the relationship between hull models and hull adjectives difficult since hull models can partially represent all hull models in the design space. If the number of sampled models that are shown in surveys is increased, it is believed that mathematical models can be computed for more hull adjectives. It is important to establish a sampling method that is appropriate to this study’s problem. Finally, infeasible hull models are possible to be obtained when the Taguchi experimental method is utilized due to the incompatibility between geometric parameter values. Extending the Taguchi method to generate only feasible hull designs is also an interesting area for future study.

4.4. Practical applications

Several applications can make use of the outcomes of this study. A large number of yacht models exist in the databases of yacht building companies. Designers generally select a design from the database and modify it to obtain the desired model. The hull adjectives determined in this work can be used to retrieve models from the database. Once customers decide the hull with a specific adjective, corresponding models in the database can be retrieved and modified. By doing this, customers and designers can collaboratively complete the design in a shorter time. Moreover, a CAD tool can be developed to generate adjective based yacht hulls from a given hull. Design variations of a hull can be achieved with a mouse click so that customers can see and comment on these designs, which can facilitate the communication between customers and designers.
5. Future works and conclusion

In this study, hull adjectives for yacht models were explored. To do this, a novel design framework with geometric parameters was first proposed for yacht hulls to quantify them. Three different survey types were introduced to capture human aesthetic shape understandings. Possible hull adjectives were collected via Survey 1. Via Survey 2, some geometric parameters were eliminated, which had no impact on the hull adjectives. The relationship between geometric parameters and hull adjectives were learned via Survey 3. The hull models in Survey 3 were obtained using the Taguchi experimental method which performs sampling from the hull design space efficiently and generates a fewer number of hulls. These hulls were shown to participants in Survey 3. At the end, mathematical models that consist of geometric parameters with coefficients were obtained, which represent hulls expressed by adjectives.

The number of hull adjectives can be increased further and new adjectives will be investigated/studied as a future work. An extra survey type (Survey 4) will be introduced to check the reliability of this study’s results. We also plan to develop an adjective based high-level CAD tool to obtain hulls having related adjectives in a shorter time. For better sampling in the hull design space, the extended Taguchi method improved by geometric constraints will be studied in order to produce only feasible hull models.

Acknowledgment

This work is supported by TUBITAK (The Scientific and Technological Research Council of Turkey - Project Number: 214M333). The authors would like to thank Mr. Shahroz Khan for his help about hulls’ hydrostatic analyses, Dr. Serkan Gunpinar for his great help and all patient responses particularly regarding the statistical analysis part of this work, Professor Hiromasa Suzuki for his valuable comments about the
Appendix

1. Hydrostatic Results of Charismatic³,Comfortable³, Cute³ and Aggressive³ models.

<table>
<thead>
<tr>
<th></th>
<th>LWL (m)</th>
<th>BWL (m)</th>
<th>V (m³)</th>
<th>Aₚ (m²)</th>
<th>LCB (m)</th>
<th>KB (m)</th>
<th>LCF (m)</th>
<th>IL (m³)</th>
<th>IT (m³)</th>
<th>Aₚ (m²)</th>
<th>BM (m)</th>
<th>KM (m)</th>
<th>Cₚ</th>
<th>Cₚ</th>
<th>Cₚ</th>
<th>Cₚ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charismatic³</td>
<td>T = 0.74</td>
<td>17.63</td>
<td>2.38</td>
<td>19.71</td>
<td>57.85</td>
<td>11.07</td>
<td>0.225</td>
<td>11.056</td>
<td>936.7</td>
<td>76.50</td>
<td>2.007</td>
<td>3.881</td>
<td>4.107</td>
<td>0.557</td>
<td>1.140</td>
<td>1.389</td>
</tr>
<tr>
<td></td>
<td>T = 1.24</td>
<td>21.47</td>
<td>2.98</td>
<td>57.41</td>
<td>89.94</td>
<td>11.033</td>
<td>0.401</td>
<td>10.941</td>
<td>2169.22</td>
<td>193.10</td>
<td>4.768</td>
<td>3.364</td>
<td>3.765</td>
<td>0.561</td>
<td>1.290</td>
<td>1.406</td>
</tr>
<tr>
<td></td>
<td>T = 1.74</td>
<td>23.16</td>
<td>3.22</td>
<td>107.66</td>
<td>109.28</td>
<td>10.935</td>
<td>0.594</td>
<td>10.766</td>
<td>3305.61</td>
<td>276.477</td>
<td>7.869</td>
<td>2.568</td>
<td>3.162</td>
<td>0.591</td>
<td>1.404</td>
<td>1.465</td>
</tr>
<tr>
<td>Comfortable³</td>
<td>T = 0.74</td>
<td>12.46</td>
<td>1.95</td>
<td>11.38</td>
<td>32.43</td>
<td>9.710</td>
<td>0.243</td>
<td>9.001</td>
<td>279.636</td>
<td>26.97</td>
<td>1.710</td>
<td>2.370</td>
<td>2.614</td>
<td>0.534</td>
<td>1.185</td>
<td>1.335</td>
</tr>
<tr>
<td></td>
<td>T = 1.24</td>
<td>17.12</td>
<td>2.58</td>
<td>34.71</td>
<td>60.74</td>
<td>8.766</td>
<td>0.395</td>
<td>7.856</td>
<td>947.705</td>
<td>95.546</td>
<td>3.791</td>
<td>2.782</td>
<td>3.177</td>
<td>0.535</td>
<td>1.185</td>
<td>1.335</td>
</tr>
<tr>
<td></td>
<td>T = 1.74</td>
<td>18.51</td>
<td>2.99</td>
<td>70.21</td>
<td>79.16</td>
<td>8.322</td>
<td>0.563</td>
<td>8.001</td>
<td>1525.79</td>
<td>169.855</td>
<td>6.547</td>
<td>2.420</td>
<td>2.983</td>
<td>0.579</td>
<td>1.258</td>
<td>1.430</td>
</tr>
<tr>
<td>Cute³</td>
<td>T = 0.74</td>
<td>18.54</td>
<td>3.2</td>
<td>10.60</td>
<td>31.58</td>
<td>8.117</td>
<td>0.224</td>
<td>7.493</td>
<td>337.158</td>
<td>19.33</td>
<td>1.323</td>
<td>1.824</td>
<td>2.045</td>
<td>0.432</td>
<td>0.559</td>
<td>0.532</td>
</tr>
<tr>
<td></td>
<td>T = 1.24</td>
<td>22.24</td>
<td>4.39</td>
<td>32.18</td>
<td>54.44</td>
<td>7.981</td>
<td>0.391</td>
<td>7.955</td>
<td>883.093</td>
<td>66.39</td>
<td>3.305</td>
<td>2.626</td>
<td>2.455</td>
<td>0.438</td>
<td>0.607</td>
<td>0.558</td>
</tr>
<tr>
<td></td>
<td>T = 1.74</td>
<td>24.45</td>
<td>4.88</td>
<td>65.09</td>
<td>74.54</td>
<td>8.05</td>
<td>0.561</td>
<td>8.292</td>
<td>1561.97</td>
<td>131.38</td>
<td>5.937</td>
<td>2.018</td>
<td>2.579</td>
<td>0.448</td>
<td>0.699</td>
<td>0.624</td>
</tr>
<tr>
<td>Aggressive³</td>
<td>T = 0.74</td>
<td>15.34</td>
<td>1.69</td>
<td>27.358</td>
<td>82.26</td>
<td>10.320</td>
<td>0.229</td>
<td>10.285</td>
<td>1414.13</td>
<td>211.107</td>
<td>2.726</td>
<td>7.716</td>
<td>7.946</td>
<td>0.654</td>
<td>2.179</td>
<td>3.173</td>
</tr>
<tr>
<td></td>
<td>T = 1.24</td>
<td>18.49</td>
<td>2.52</td>
<td>82.806</td>
<td>136.13</td>
<td>10.436</td>
<td>0.395</td>
<td>10.727</td>
<td>3525.08</td>
<td>628.168</td>
<td>6.617</td>
<td>7.586</td>
<td>7.981</td>
<td>0.677</td>
<td>2.118</td>
<td>2.922</td>
</tr>
<tr>
<td></td>
<td>T = 1.74</td>
<td>19.74</td>
<td>2.78</td>
<td>158.248</td>
<td>161.236</td>
<td>10.696</td>
<td>0.584</td>
<td>11.148</td>
<td>5104.71</td>
<td>851.116</td>
<td>6.547</td>
<td>2.420</td>
<td>2.983</td>
<td>0.579</td>
<td>1.258</td>
<td>1.430</td>
</tr>
</tbody>
</table>


Appendix A. Supplementary data


